

AP[®] Calculus Existence Theorems & Formulas

College Board CED Aligned · Core FRQ Justification Templates

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1. THE BIG 4 EXISTENCE THEOREMS (MANDATORY FRQ JUSTIFICATIONS)

Theorem	Prerequisites	Mathematical Conclusion	Official AP [®] Grader Justification Template
IVT (Intermediate Value Theorem)	f is continuous on [a, b]	For any k between f(a) and f(b), there exists at least one c in (a, b) such that $f(c) = k$.	"Since f is continuous on [a, b], and $f(a) < k < f(b)$, by the Intermediate Value Theorem there exists c in (a, b) such that $f(c) = k$."
MVT (Mean Value Theorem)	1. Continuous on [a, b] 2. Differentiable on (a, b)	There exists at least one c in (a, b) such that $f'(c) = (f(b) - f(a)) / (b - a)$.	"Since f is continuous on [a, b] and differentiable on (a, b), by the Mean Value Theorem there exists a value c where instantaneous rate equals average rate."
EVT (Extreme Value Theorem)	f is continuous on closed interval [a, b]	f attains both an absolute maximum and an absolute minimum on [a, b].	"Candidates Test: Evaluate f(x) at all critical points ($f'(x) = 0$ or undefined) and at interval endpoints $x = a$, $x = b$."
FTC Part 1 & 2 (Fundamental Theorem)	f is continuous on [a, b]	$\frac{d}{dx} \int[a \text{ to } x] f(t)dt = f(x)$ $\int[a \text{ to } b] f'(x)dx = f(b) - f(a)$	" $f(b) = f(a) + \int[a \text{ to } b] f'(t)dt$. The net change in f equals the definite integral of its rate of change."

2. HIGH-YIELD DERIVATIVE & INTEGRAL RULES

Operation	Standard Form	Chain Rule Extended Form
Product Rule	$\frac{d}{dx} [u \cdot v] = u'v + uv'$	Keep first, differentiate second + keep second, differentiate first
Quotient Rule	$\frac{d}{dx} [u / v] = (u'v - uv') / v^2$	"Low d-High minus High d-Low, over the square of what's below"
Trig Derivatives	$\frac{d}{dx} \sin(x) = \cos(x)$ $\frac{d}{dx} \cos(x) = -\sin(x)$	$\frac{d}{dx} \tan(x) = \sec^2(x)$ $\frac{d}{dx} \sec(x) = \sec(x)\tan(x)$

Operation	Standard Form	Chain Rule Extended Form
Inverse Trig	$\frac{d}{dx} \arcsin(x) = 1/\sqrt{1-x^2}$	$\frac{d}{dx} \arctan(x) = 1/(1+x^2)$
Exponential & Log	$\frac{d}{dx} e^{(kx)} = k \cdot e^{(kx)}$	$\frac{d}{dx} \ln x = 1/x, \frac{d}{dx} a^x = a^x \cdot \ln(a)$

3. JUSTIFYING LOCAL EXTREMA & INFLECTION POINTS (FRQ POINTS)

First Derivative Test for Local Max/Min:

"f has a local maximum at $x = c$ because $f'(c) = 0$ (or undefined) and $f'(x)$ changes sign from positive to negative at $x = c$."

Point of Inflection Justification:

"The graph of f has a point of inflection at $x = c$ because $f''(x)$ changes sign (from positive to negative or negative to positive) at $x = c$."

 **FRQ Warning:** Never say "the graph changes from increasing to decreasing" or "it changes direction." Graders require explicitly naming $f'(x)$ or $f''(x)$.